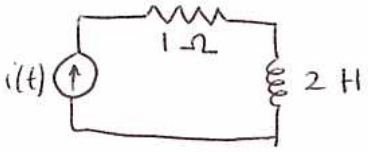
BME 153.02 Homework #5

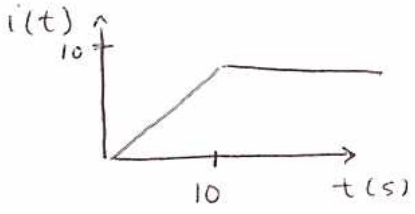
Solutions

4.4

$$i(t) = \begin{cases} 0 & \text{for } -\infty < t < 0 \\ t & \text{for } 0 \le t < 10\,s \\ 10 & \text{for } 10\,s \le t < \infty \end{cases}$$



Find energy stored in inductor for all time



$$W_L(t) = \frac{1}{2} L\, i(t)^2$$

$$W_L(t) = \begin{cases} 0 & -\infty < t < 0 \\ t^2 & 0 \le t < 10\,s \\ 100\,J & 10\,s \le t < \infty \end{cases}$$

4.5

Find energy delivered by the source at all time

$$W_S(t) = W_L(t) + W_R(t)$$

$$\hookrightarrow W_R(t) = \int P_R(t)\,dt \Rightarrow$$

$$\hookrightarrow P_R(t) = i_R(t)^2 R = \begin{cases} 0 & -\infty < t \\ t^2 & 0 \le t \\ 100\,W & 10 \le \end{cases}$$

$$W_R(t) = \int P_R(t)\,dt$$

$$\hookrightarrow \int 0\,dt = 0$$

$$\hookrightarrow \int t^2\,dt = \frac{t^3}{3}$$

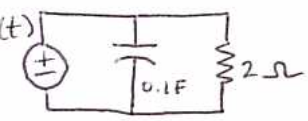
$$\hookrightarrow \int_0^{10} t^2\,dt + \int_{10}^{t} 100\,dt = \frac{10^3}{3} + 100(t-10)$$

$$\Rightarrow W_R(t) = \begin{cases} 0 & -\infty < \\ t^3/3 & 0 \le \\ \frac{1000}{3} + 100(t-10) & 10 \le \end{cases}$$

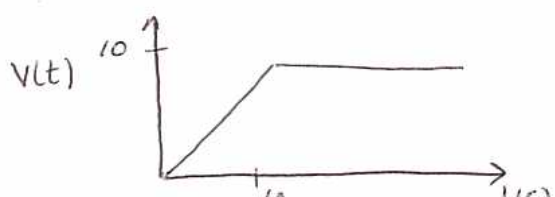
$$W_S(t) = W_L(t) + W_R(t) = \begin{cases} 0 & -\infty < t < 0 \\ t^3/3 + t^2 & 0 \le t < 10\,s \\ \frac{1000}{3} + 100(t-10) + 100 & 10\,s \le t < \infty \end{cases}$$

4.7

Find energy stored in capacitor



$$V(t) = \begin{cases} 0 & -\infty < t < 0 \\ t & 0 \le t < 10\,s \\ 10 & 10\,s \le t < \infty \end{cases}$$



$$W_C(t) = \frac{1}{2} C V_C^2$$

$$W_C(t) = \begin{cases} 0 & -\infty < t < 0 \\ 0.05t^2 & 0 \le t < 10 \\ 5 & 10 \le t < \infty \end{cases}$$

4.8 Find energy delivered to Source

$$W_S(t) = W_C(t) + W_R(t)$$

$$\hookrightarrow W_R(t) = \int P_R(t)\,dt$$

$$\hookrightarrow P_{R(t)} = \frac{V_R^2}{R} = \begin{cases} 0 & -\infty < t < 0 \\ t^2/2 & 0 \le t < 10s \\ 50\,W & 10s \le t < \infty \end{cases}$$

$$W_R(t) = \int P_R(t)\,dt$$

$$\hookrightarrow \int 0\,dt = 0$$

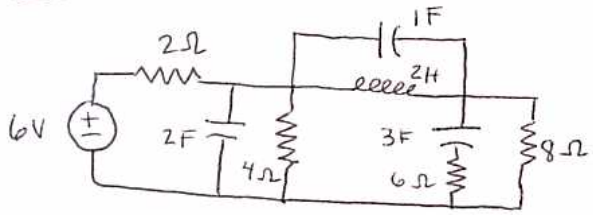
$$\hookrightarrow \int t^2/2\,dt = \frac{t^3}{6}$$

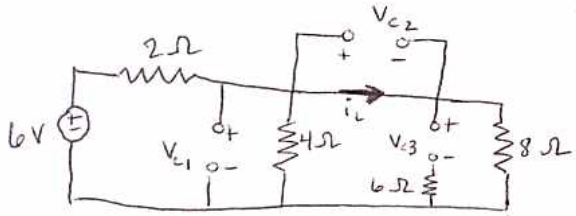
$$\hookrightarrow \int_0^{10} \frac{t^2}{2}\,dt + \int_{10}^{t} 50\,dt = \frac{10^3}{6} + 50(t-10)$$

$$\Rightarrow W_R(t) = \begin{cases} 0 & -\infty < t < \\ t^3/6 & 0 \le t < \\ \frac{10^3}{6} + 50(t-10) & 10s \le t < \end{cases}$$

$$W_S(t) = W_C(t) + W_R(t) = \begin{cases} 0 & -\infty < t < 0 \\ \frac{t^2}{20} + \frac{t^3}{6} & 0 \le t < 10s \\ \frac{515}{3} + 50(t-10) & 10s \le t < \infty \end{cases}$$

4.10





KCL: $\frac{6 - V_A}{2} = \frac{V_A}{4} + \frac{V_A}{8} \Rightarrow V_A = \frac{24}{7}\,V$

$V_{C1} = V_A = \frac{24}{7}\,V$

$V_{C2} = 0\,V$

$V_{C3} = V_A = \frac{24}{7}\,V$

$i_L = \frac{V_A}{8} = \frac{3}{7}\,A$

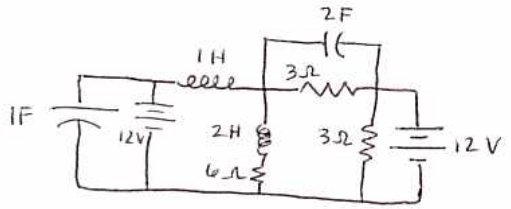
$W_L = \frac{1}{2} L i_L^2 = \frac{9}{49}\,J \Rightarrow W_L = 0.18\,J$

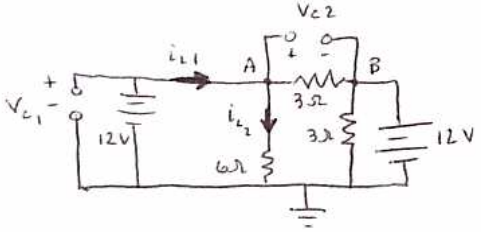
$W_{C_1} = \frac{1}{2} C V_{C_1}^2 \Rightarrow W_{C_1} = 11.76\,J$

$W_{C_2} = \frac{1}{2} C V_{C_2}^2 \Rightarrow W_{C2} = 0\,J$

$W_{C_3} = \frac{1}{2} C V_{C_3}^2 \Rightarrow W_{C3} = 17.6\,J$

4.11





$V_A = V_B = 12\,V$

$V_{C1} = 12\,V$

$V_{C2} = V_A - V_B = 0\,V$

$i_{L2} = \frac{V_A}{6\Omega} = 2\,A$

$i_{L1} = \frac{V_A - V_B}{3} + i_{L_2} = 2\,A$

$\Rightarrow$

$W_{C_1} = \frac{1}{2} C_1 V_{C_1}^2 \Rightarrow W_{C_1} = 72\,J$

$W_{C_2} = \frac{1}{2} C_2 V_{C_2}^2 \Rightarrow W_{C_2} = 0\,J$

$W_{L_1} = \frac{1}{2} L_1 i_{L_1}^2 \Rightarrow W_{L1} = 2\,J$

$W_{L_2} = \frac{1}{2} L_2 i_{L_2}^2 \Rightarrow W_{L2} = 4\,J$