 Name: Answer Key

BME153L.02 (Palmeri)

Spring 2009

Test #2: Dynamic Circuit Elements and Filters

Instructions:

- Write your name at the top of each page.
- Show all work (this is *critical* for partial credit!).
- Represent all time-domain functions as such; do not leave answers in phasor notation.
- Only work in the space provided. Ask for extra paper if necessary.
- Read through each complete question before starting to work (this may save you some time).
- Remember to include units with all answers and label all plot axes.
- Clearly box all answers.

In keeping with the Duke Community Standard, I have neither given nor received aid in completion of this examination.

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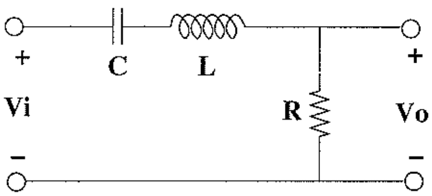
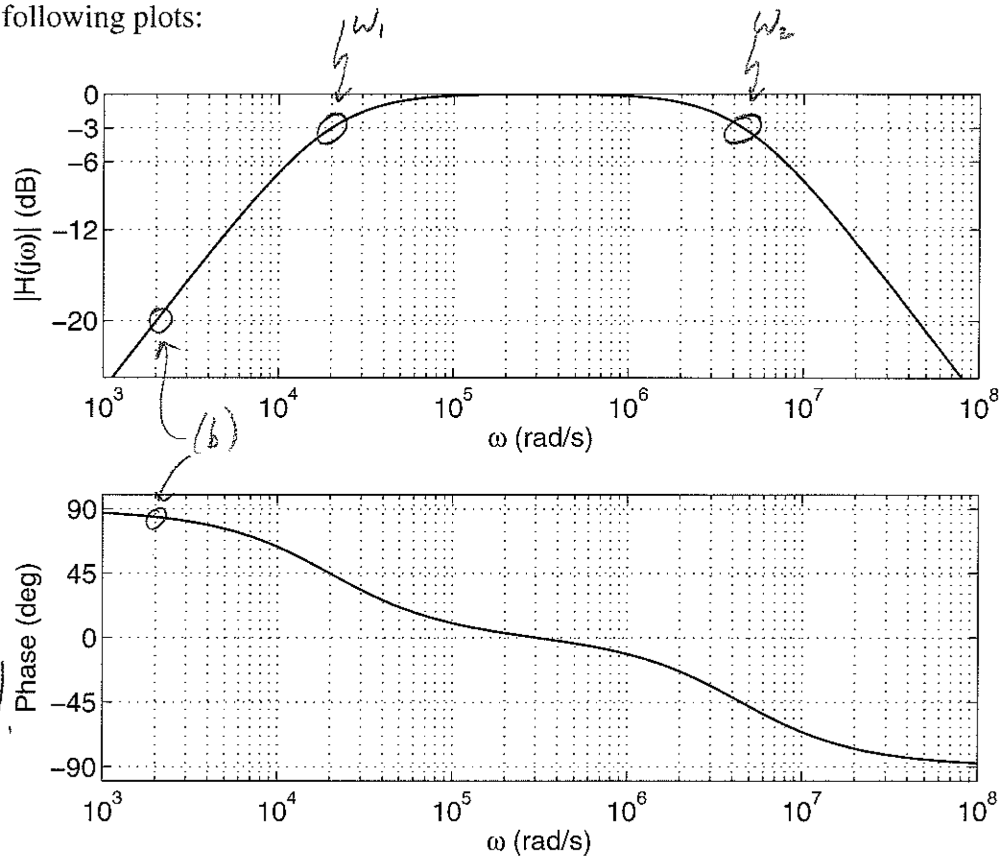
Problem #1 [20 points] $\left(\frac{15.4}{20}\right)$

You're given the following filter, and you measure the magnitude and phase of its transfer function ($H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$) in the lab, as shown in the following plots:



Code used to generate these plots attached.



(a) What type of filter is this (specify whether it is first- or second-order)? What is/are this filter's cutoff frequency/frequencies (reasonable estimates are okay)? [5 points]

BPF (2°)

$$\left.\begin{aligned} \omega_1 &\approx 2\times10^4 \text{ rad/s} \\ \omega_2 &\approx 4.5\times10^6 \text{ rad/s} \end{aligned}\right\} \text{-3dB points}$$

(b) If $V_i(t) = 0.5\cos(2000t)$ V, then what is $V_o(t)$ (again, reasonable estimates are okay)? [5 points]

$$-20\text{dB} = 20\log_{10}\left(\frac{|V_o|}{|V_i|}\right)\text{dB}$$

$$-1 = \log_{10}\left(\frac{|V_o|}{0.5}\right)$$

$$|V_o| = 0.05$$

$$\phi\Big|_{\omega=2000\text{rad/s}} \approx 85°$$

$$\boxed{V_o(t) = 0.05\cos(2000t + 85°)\text{ V}}$$

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(c) Solve for the values of R & L needed to achieve this transfer function, assuming that C = 5.55 nF and the quality factor for this circuit can be expressed as $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. [10 points]

$$\omega_n = 3\times10^5 \text{ rad/s} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L\times1\times10^{-9}F)}} \rightarrow \boxed{L = 2mH}$$

$$Q = \frac{\omega_n}{B} = \frac{\omega_n}{\omega_2-\omega_1} = \frac{3\times10^5}{(4.5\times10^6 - 2\times10^4)} = 0.067$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 0.067 = \frac{1}{R}\sqrt{\frac{2\times10^{-3}}{5.55\times10^{-9}}}$$

$$\boxed{R \approx 9k\Omega}$$

```
% BME153L.02 (Spring 2009)
% Test #2 BPF Transfer Function Interpretation
% Mark Palmeri (mark.palmeri@duke.edu)
% 2009-03-21

R = 9000;
C = 5.55e-9;
L = 2e-3;

w = logspace(3,8,10000);

H = (i*w*C*R)./(1+i*w*C*R+((i*w).^2)*L*C);

subplot(2,1,1);
h=semilogx(w,20*log10(abs(H)));
set(h,'LineWidth',1,'Color','k');
xlabel('\omega (rad/s)','FontSize',14)
ylabel('|H(j\omega)| (dB)','FontSize',14)
set(gca,'YTick',[-40 -20 -12 -6 -3 0])
set(gca,'FontSize',14);
grid on;
a=axis;
axis([a(1) a(2) -25 0]);

subplot(2,1,2);
h=semilogx(w,angle(H)*360/(2*pi));
set(h,'LineWidth',1,'Color','k');
xlabel('\omega (rad/s)','FontSize',14)
ylabel('Phase (deg)','FontSize',14)
set(gca,'YTick',[-90:45:90]);
grid on;
set(gca,'FontSize',14);

print -deps2 bpf.eps
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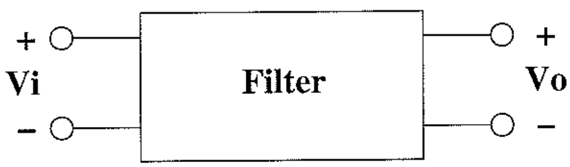
Problem #2 [35 points] $\left(\frac{28.4}{35}\right)$



You are designing a filter for a signal coming out of an ultrasound scanner. The desired signal content ranges in frequency from 1-10 MHz, but there is significant noise $\leq$ 100 Hz. You must design a <u>first-order</u> filter that:

- Attenuates noise $\leq$ 100 Hz at least 40 dB below the desired signal content (1-10 MHz),
- Does not distort the phase of the desired signal content (1-10 MHz) more than 15°.

(a) What type of filter do you need? Why? [5 points]

HPF; need to preserve high f content while attenuating low f noise.
Can't use BPF b/c it is 2°.

(b) What is/are the cutoff frequency/frequencies of your filter? [5 points]

1° HPF attenuates −20 dB/decade below ω_c + phase distortion $\rightarrow 0$ ~1 decade above ω_c

$$\therefore f_c \left\{ \leq 1\times10^5\,Hz + \geq 1\times10^4\,Hz \right. \rightarrow 5\times10^4\,Hz$$

$$\boxed{\omega_c = 2\pi(5\times10^4\,Hz) = 3.14\times10^5\,rad/s}$$

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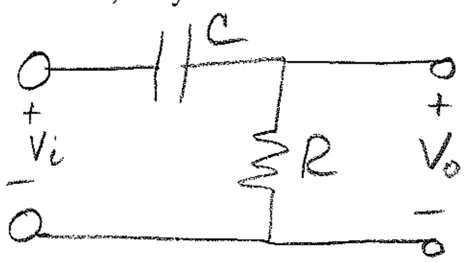
(c) Using 1 kΩ resistors and any value capacitors and/or inductors, draw a circuit diagram, including component values, for your filter that achieves the objectives described above. [5 points]



$R = 1k\Omega$

$w_c = 3.14 \times 10^5 \, rad/s$

$w_c = \frac{1}{RC} \longrightarrow 3.14 \times 10^5 = \frac{1}{(1 \times 10^3)C}$

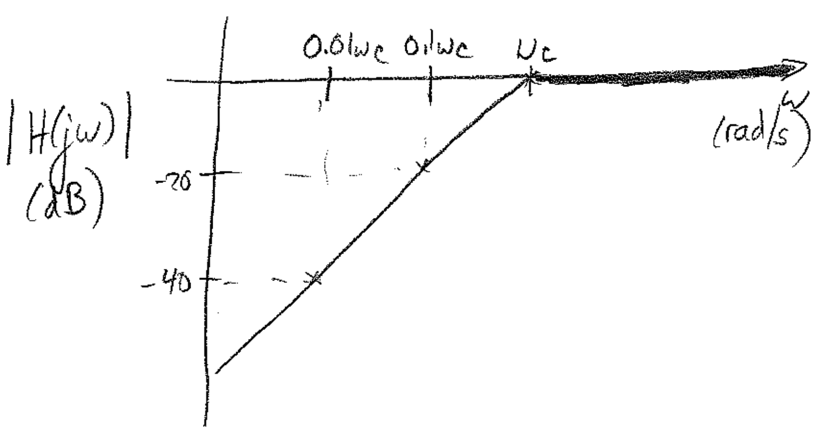
$\boxed{C = 3.185 \, nF}$

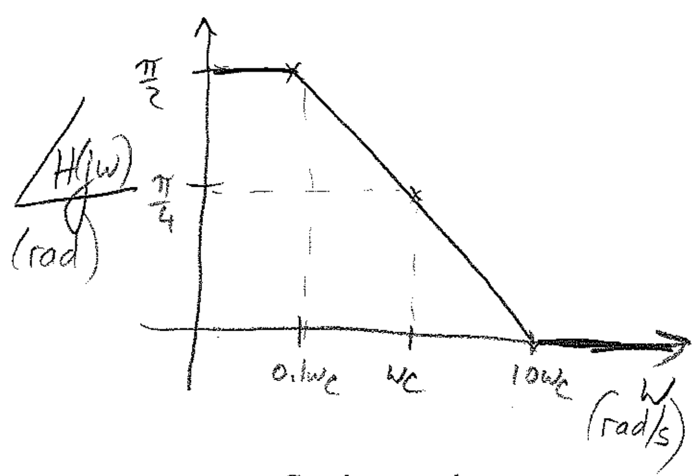
(d) Derive (don't just state) the transfer function ($H(j\omega)$) for your filter. [5 points]

$$V_o(jw) = V_i(jw) \frac{R}{R + \frac{1}{jwC}}$$

$$H(jw) = \frac{V_o(jw)}{V_i(jw)} = \frac{jwRC}{1 + jwRC} = \boxed{\frac{wRC}{\sqrt{1 + (wRC)^2}} \angle \frac{\pi}{2} - \tan^{-1}(wRC)}$$

(e) Draw the Bode plot for your filter, including both magnitude and phase, labeling all important features and axes. [5 points]





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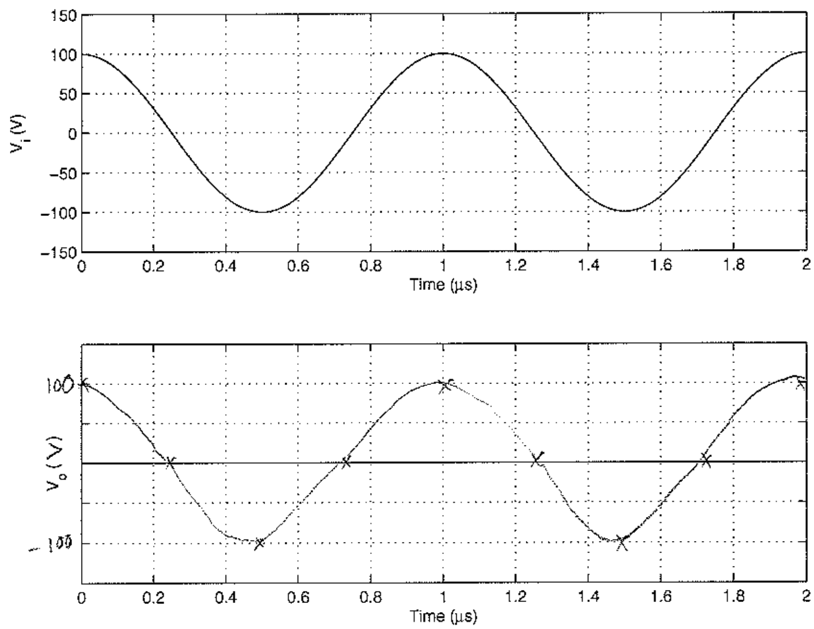
(f) If your input signal is $V_i(t) = 100\cos(2e6\pi t)$ V, then what is the filter's output ($V_o(t)$)? In addition to writing the expression for $V_o(t)$, plot $V_o(t)$ below the plot of $V_i(t)$. Remember to label the voltage axis. (Check yourself - this input signal should be in the passband of your filter - remember the stated filter objectives.) [5 points]

Using Bode plot or transfer function...

$V_o(t) = 100\cos(2e6\pi t + 2.9°)$ V ✓

not attenuated ✓

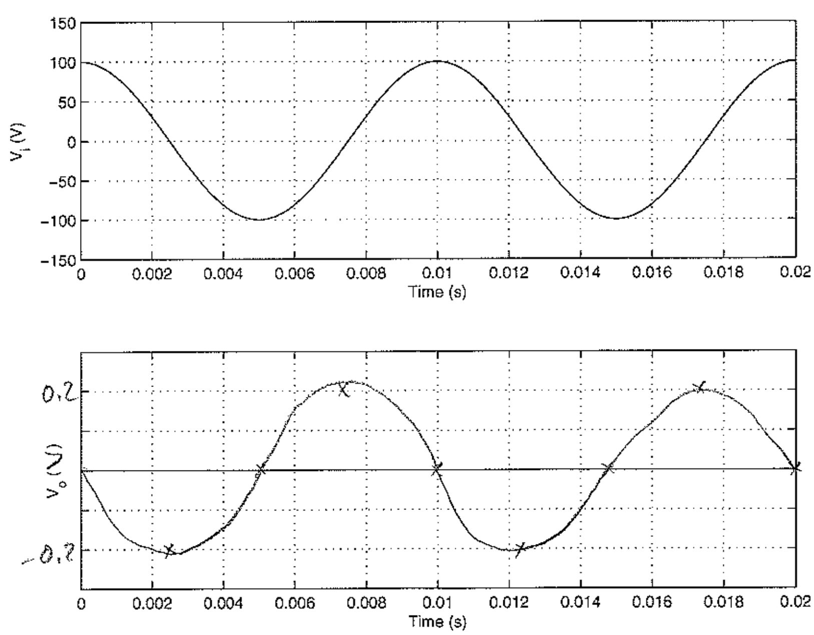
$< 15°$ ✓



(g) Repeat (f) for $V_i(t) = 100\cos(200\pi t)$ V. (Check yourself - this input signal should be in the noise band - remember the stated filter objectives.) [5 points]

$V_o(t) = 0.2\cos(200\pi t + 90°)$

< -49dB ✓



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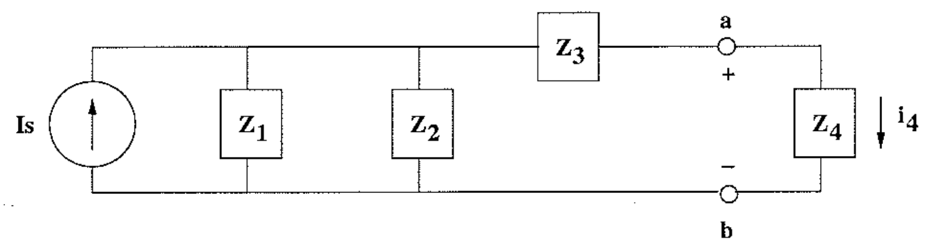
Problem #3 [25 points] $\left(\frac{19.3}{25}\right)$



$I_s(t) = 10\cos(1000t)$ mA
$Z_1 = 20\ \Omega$
$Z_2 = 30\ \Omega$
$Z_3 = ?$
Z_4 = variable load

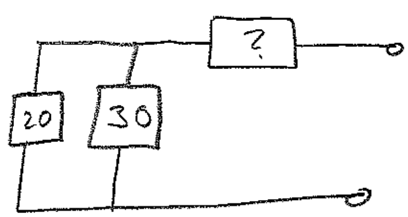
$Z_{1..4}$ represent discrete components in the circuit (i.e., resistors, capacitors, and inductors). When $Z_4 = \infty$ (open circuit across terminals a & b), $v_{ab} = 120\cos(1000t)$ mV, and when $Z_4 = 0$ (short circuit across terminals a to b), $i_4 = 9.86\cos(1000t + 9.46°)$ mA.

(a) What is the Norton impedance for the circuit as seen from Z_4 (i.e., Z_4 acts as the load)? [5 points]

$$Z_n = \frac{V_{oc}}{i_{sc}} = \frac{120\times10^{-3}}{9.86\times10^{-3}\angle 9.46°} = 12.17\angle{-9.46°}\ \Omega$$

$$= \boxed{12.0 - 2j\ \Omega}$$

(b) Given Z_T, what is Z_3? [5 points]



$$12 - 2j = Z_3 + 12$$

$$\boxed{Z_3 = -2j\,\Omega}$$

(c) Is Z_3 a resistor, a capacitor, or an inductor? What is the value of this component? [5 points]

Capacitor (−90° phase shift)

$$Z_3 = -2j = \frac{-j}{\omega C}\Big|_{\omega = 1000\,rad/s} \longrightarrow \boxed{C = 500\,\mu F}$$

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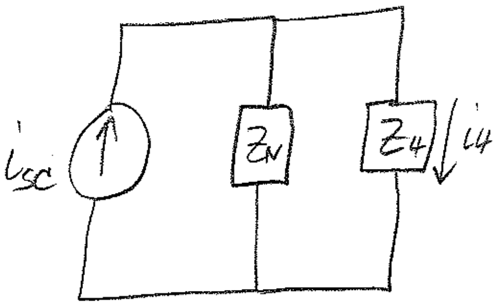
(d) Solve for $i_4(t)$ if Z_4 is a 5 mH inductor. [5 points]



$$Z_4 = j\omega L = 5j\ \Omega$$

$$i_4 = 9.86\angle 9.46^\circ \ \frac{12.17\angle -9.46^\circ}{12-2j+5j}\ mA$$

$$i_4 = \frac{120}{12-2j+5j} = \frac{120}{12+3j} = 9.7\angle -14^\circ\ mA$$

$$\boxed{i_4(t) = 9.7\cos(1000t-14^\circ)\,mA}$$

(e) Solve for $v_{ab}(t)$ if Z_4 is a 5 mH inductor (same as (d)). [5 points]

$$v_{ab}(t) = v_4(t) = L\frac{di_4(t)}{dt} = (5\times10^{-3})\cancel{(-1000)}\cancel{(10^{-3})}(9.7)\sin(1000t-14^\circ)\,V$$

$$\boxed{v_{ab}(t) = -48.5\sin(1000t-14^\circ)\,mV}$$

$$\boxed{v_{ab}(t) = 48.5\cos(1000t+76^\circ)\,mV}$$

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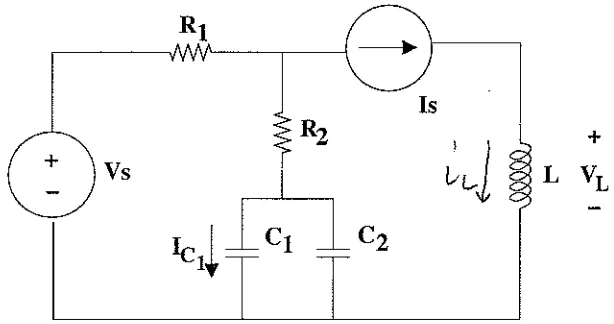
Problem #4 [~~15~~ 20 points] $\frac{10.8}{20}$



$V_s(t) = 20\cos(100t)$ V
$I_s(t) = 10\cos(1000t + 20°)$ mA
$R_1 = 100\ \Omega$
$R_2 = 500\ \Omega$
$C_1 = 250\ \mu F$
$C_2 = 750\ \mu F$
$L = 5$ mH

Assume that all of the sources have been on for a long time (i.e., the circuit is in a steady-state condition).

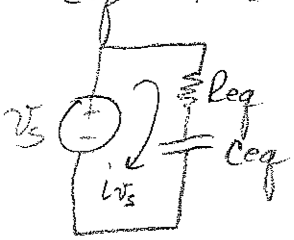
(a) Solve for an expression for $V_L(t)$. [5 points]

$$v_L = i_L Z_L = (10\angle 20°)(j\,1000 \times 5 \times 10^{-3}) = 50\angle 110°$$

$$\boxed{v_L(t) = 50\cos(1000t + 110°)\ mV}$$

(b) Solve for an expression for $I_{C_1}(t)$. [15 points] *Using superposition...*

$C_{eq} = C_1 + C_2 = 1000\,\mu F$



$R_{eq} = 600\,\Omega$

$$Z_{C_{eq}} = \frac{-j}{\omega C} = -10j$$

$$i_{v_s} = \frac{V_s}{Z_{eq} + Z_{C_{eq}}} = \frac{20\angle 0}{600 - 10j} = 0.0333\angle 1.0°$$

Current Division w/i C_{eq}: $i_{C_1 v_s} = i_{v_s}\frac{Z_{C_1}}{Z_{C_1} + Z_{C_2}} = 0.0333\angle 1.0° \; \frac{\overbrace{-13.33j}^{0.25}}{-40j - 13.33j}$

$$\boxed{i_{C_1 v_s}(t) = 0.0083\cos(100t + 1.0°)\,A}$$

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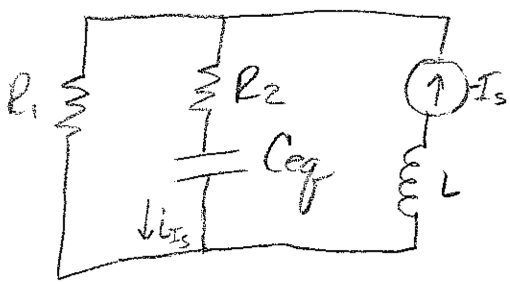
Problem #4 continued...



$$i_{I_s} = -I_s \frac{100}{100 + 500 - 10j}$$

$$= -10\angle 20^\circ \frac{100}{600 - j}$$

$$= \frac{-10\angle 20^\circ}{600\angle 0.1^\circ}(100) = -\frac{5}{3}\angle 19.9^\circ$$

$$Z_{eq} = \frac{-j}{\omega C} = -j$$

$$i_{C_1 I_s} = i_{I_s}(0.25) = -\frac{5}{12}\angle 19.9$$

$$i_{C_1 I_s}(t) = \left(-\frac{5}{12}\cdot 10^3\right)\cos(1000t + 19.9^\circ)\,A$$

$$\boxed{i_{C_1}(t) = 8.3\cos(100t + 1.0^\circ) - 0.417\cos(1000t + 19.9^\circ)\,mA}$$