Name: Answer Key

BME153L.02 (Palmeri)

Spring 2009

Test #1: Resistive Circuit Analysis

Instructions:

- Place your name at the top of each page.
- All sources are assumed to be ideal and linear, and all resistive elements are also assumed to be linear.
- Show all work
  - Clearly draw and label all circuit diagrams during your analysis, including element polarity, current directions, and node and mesh current labels when appropriate.
  - Redraw circuits with labels when you've reduced the circuit to equivalent elements/networks.
  - Indicate your analysis method you are using when setting up your equations (e.g., KVL around loop 1, KCL at node A, etc.).
- Only work in the space provided. Use the backs of pages if necessary, but clearly indicate which questions continue onto the back.
- Read through each complete question before starting to work on earlier parts (this may save you some time).
- Clearly box all answers.

In keeping with the Duke Community Standard, I have neither given nor received aid in completion of this examination.

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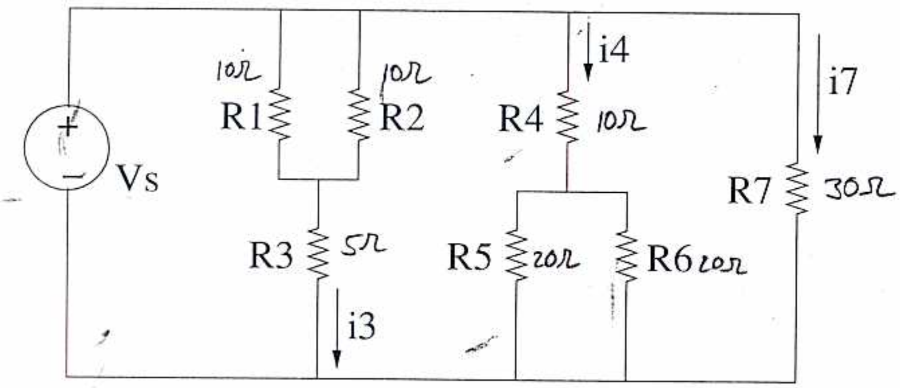
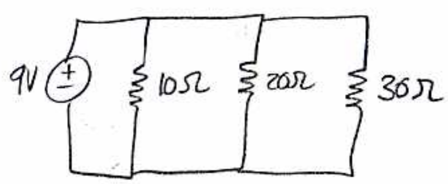
1 Where do the electrons flow?

(a) Solve for the currents in each of the branches (i_3, i_4, i_7) shown in the following circuit shown in Figure 1.1. Use any analysis method you would like, but clearly indicate which analysis method you are using with appropriate circuit diagrams/labels and equations. [5 points]



Component	Value
V_s	9 V
R_1	10 Ω
R_2	10 Ω
R_3	5 Ω
R_4	10 Ω
R_5	20 Ω
R_6	20 Ω
R_7	30 Ω

Figure 1.1: Voltage Source Circuit



$$i_3 = \frac{9V}{10\Omega} = 0.9A$$
$$i_4 = \frac{9V}{20\Omega} = 0.45A$$
$$i_7 = \frac{9V}{30\Omega} = 0.3A$$

How much power is supplied by the voltage source (magnitude only)? [5 points]

$$P_{V_S} = (9V)(0.9+0.45+0.3)A = \boxed{14.85W}$$

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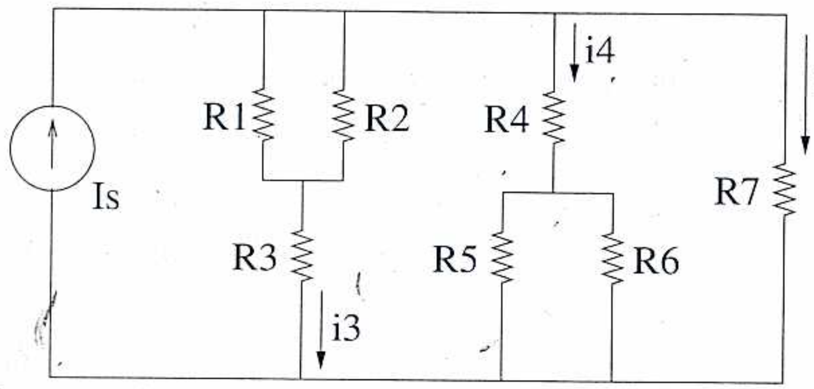
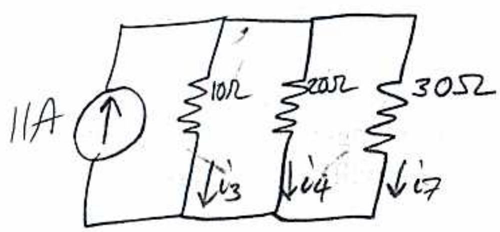
(b) Solve for the new branch currents (i_3, i_4, i_7) when a current source is substituted for the voltage source: [5 points]



Component	Value
I_s	11 A
R_1	10 Ω
R_2	10 Ω
R_3	5 Ω
R_4	10 Ω
R_5	20 Ω
R_6	20 Ω
R_7	30 Ω

Figure 1.2: Current Source Circuit



$$\boxed{\begin{aligned} i_3 &= (11A)\frac{0.1}{0.183} = 6.0A \\ i_4 &= (11A)\frac{0.05}{0.183} = 3.0A \\ i_7 &= (11A)\frac{0.033}{0.183} = 2.0A \end{aligned}}$$

$G_1 = 0.1S$

$G_2 = 0.05S$

$G_3 = 0.033S$

$\sum_{n=1}^{3} G_n = 0.183S$

$i_3 + i_4 + i_7 = 11A = I_S$ ✓

How much power is supplied by the current source (magnitude only)? [5 points]

$P_{I_S} = (6.0A)(10\Omega)(11A) = \boxed{660W}$

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2 If I only had a voltage source...

It is the night before a big senior design project is due, and you need to build the following circuit that can be attached to an arbitrary load across the output terminals:

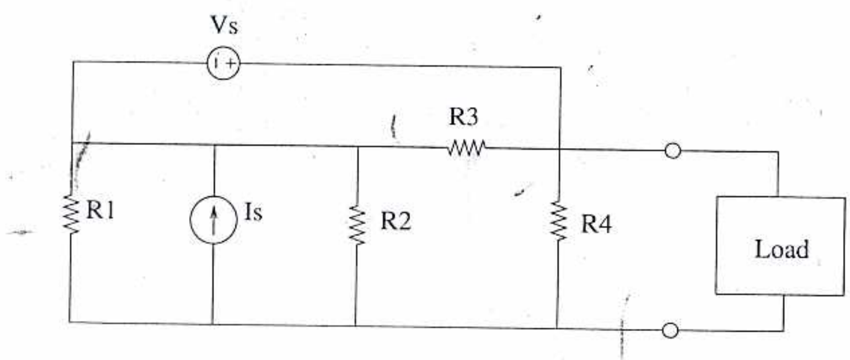


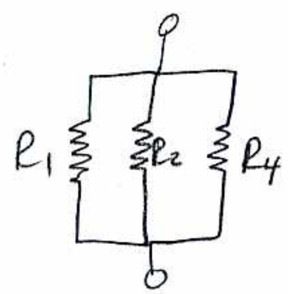
Figure 2.1: Big senior project circuit

Component	Value
V_s	5 V
I_s	1 A
R_1	5 Ω
R_2	10 Ω
R_3	10 Ω
R_4	20 Ω

Unfortunately the other members of your group dropped the ball and only bought a single ideal current source. At first you panic, but then you realize that you learned some tricks in BME153L that will save the day.

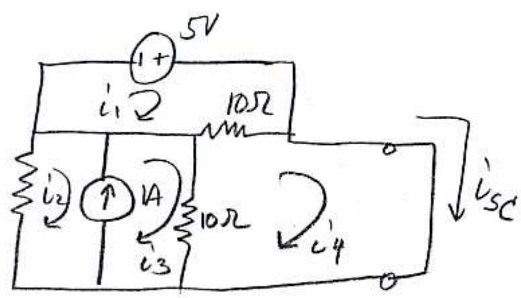
(a) Construct the Norton equivalent circuit for the circuit shown in Figure 2.1. Clearly solve for, draw and label the Norton equivalent circuit using **mesh current analysis**. How much current does the current source need to provide, and what value resistor do you need for your circuit? [15 points]

R_N :



$\rightarrow R_N = \left[\frac{1}{5} + \frac{1}{10} + \frac{1}{20}\right]^{-1} \Omega = 2.857\Omega$

i_{sc} :



$KVL_{①}: -5 + (i_1 - i_4)10 = 0 \rightarrow 10i_1 - 10i_4 = 5$

$i_1 = 0.5 + i_4$

$KVL_{②}: i_2 - i_3 = -1A$

$KVL_{④}: (i_4 - i_3)10\Omega + (i_4 - i_1)10\Omega = 0$

$-10i_1 - 10i_3 + 20i_4 = 0$

$i_1 + i_3 - 2i_4 = 0$

$KVL_{(2+4)}: i_2(5) - 5 = 0$

$5i_2 = 5 \rightarrow i_2 = 1A$

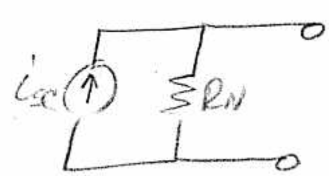
$i_3 = 1 + 1 = 2A$

$i_4 = i_{sc}$

$0.5 + i_4 + 2 - 2i_4 = 0$

$-i_4 = -2.5$

$i_4 = \boxed{2.5A = i_{sc}}$

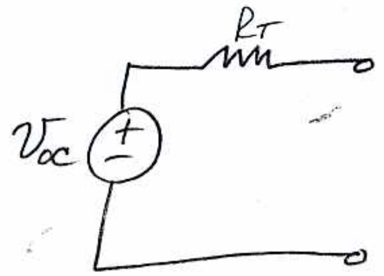


(b) After building your Norton equivalent circuit, you realize that the current source that your group ordered is a dud. Just as you make this realization, a friend walks by the lab and offers you an ideal voltage source that you can use for your project. Clearly solve for, draw and label the Thévenin equivalent circuit for the circuit shown in Figure 2.1. How much voltage does the voltage source need to provide, and what value resistor do you need for your circuit? [10 points]

$V_{oc} = R_N i_{sc} = (2.875\Omega)(2.5A)$

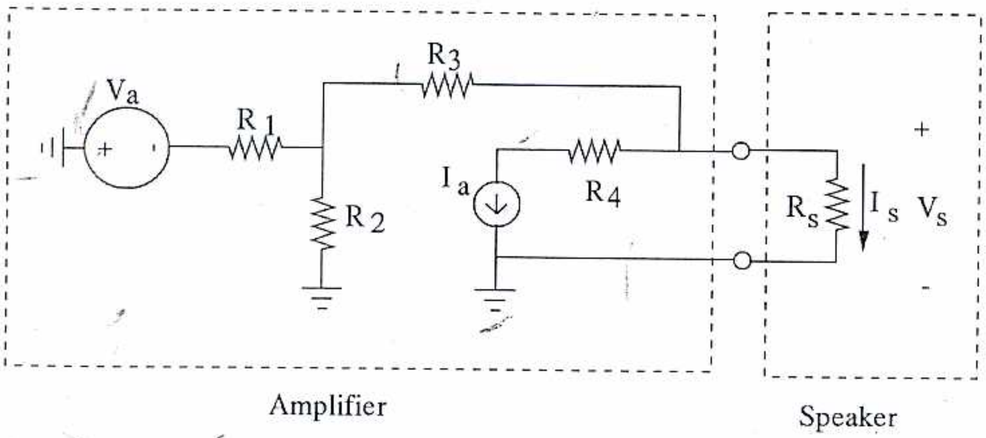
$\boxed{V_{oc} = 7.1875\,V}$

$\boxed{R_T = R_N = 2.857\Omega}$



3 How loud will this get?!

After a long Thursday BME153 lab, you decide to go back to your dorm to setup your new stereo system. You bought one amplifier, but you weren't sure what speakers to get, so you have 4 different models on hand. The figure below shows a simplified circuit schematic for your stereo amplifier and attached speaker.



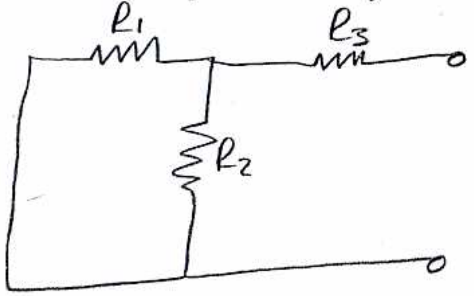
Component	Value
V_a	10 V
I_a	5 A
R_1	6 Ω
R_2	6 Ω
R_3	5 Ω
R_4	3 Ω

Figure 3.1: Amplifier/Speaker Stereo Circuit

You pull out your handy ohmmeter in your dorm room an measure the resistance of the 4 speakers that you bought, and you come up with the following values: $R_{S1} = 4\ \Omega$, $R_{S2} = 8\ \Omega$, $R_{S3} = 12\ \Omega$, $R_{S4} = 16\ \Omega$.

Clearly demonstrate the use of **node voltage analysis** to solve for any voltage or current values in your circuit analysis in parts (a) and/or (b).

(a) Given the amplifier circuit shown in Figure 3.1, what would be the ideal speaker resistance (R_{Si}) to achieve maximim power delivery from the amplifier? [10 points]

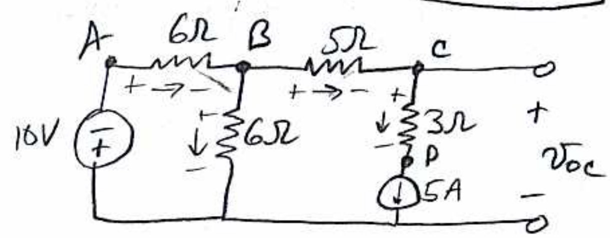


$$\boxed{R_T = 5\Omega + 3\Omega = 8\Omega = R_{Si}}$$

(b) Complete the following table describing the current (I_s), voltage V_s, and power (P_s) that would be delivered to each speaker if they were attached to the amplifier circuit as shown in Figure 3.1. (Note - V_s and I_s are the voltage and current associated with the speaker, not the sources in the amplifier circuit!) As with all problems, show all of your work! [15 points]

Speaker	I_s (A)	V_s (V)	P_s (W)
$R_{S1} = 4\,\Omega$	-3.75	-15	56.25
$R_{S2} = 8\,\Omega$	-2.8	-22.5	63.28 ✓
$R_{S3} = 12\,\Omega$	-2.25	-27	60.75
$R_{S4} = 16\,\Omega$	-1.875	-30	56.25

Based on your table, which speaker allows for the greatest power delivery? Does this agree with your answer from part (a)? $\boxed{R_{S_2} = 8\Omega}$ Yes!



$v_C = v_{oc}$

$v_A = -10V$

$KCL_B: \dfrac{v_A - v_B}{6} = \dfrac{v_B}{6} + \dfrac{v_B - v_C}{5}$

$-\dfrac{10}{6} - \dfrac{v_B}{6} = \dfrac{v_B}{6} + \dfrac{v_B}{5} - \dfrac{v_C}{5}$

$\dfrac{16}{30} v_B - \dfrac{1}{5} v_C = -\dfrac{10}{6}$

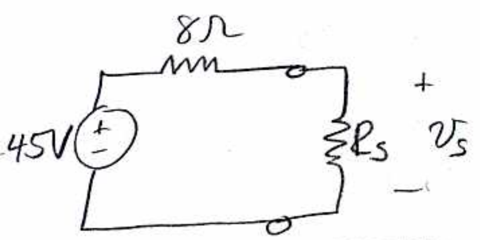
$KCL_C: \dfrac{v_B - v_C}{5} = 5$

$v_B = v_C + 25$

$\dfrac{16}{30}(v_C + 25) - \dfrac{v_C}{5} = -\dfrac{10}{6}$

$\dfrac{1}{3} v_C \cdot -15$

$v_C = \boxed{-45V = v_{oc}}$



$\boxed{v_s = -45 \dfrac{R_s}{R_s + 8}}$

$\boxed{P_s = \dfrac{v_s^2}{R_s} = \dfrac{2025 R_s}{(R_s + 8)^2}}$

$\boxed{i_s = \dfrac{P_s}{v_s}}$

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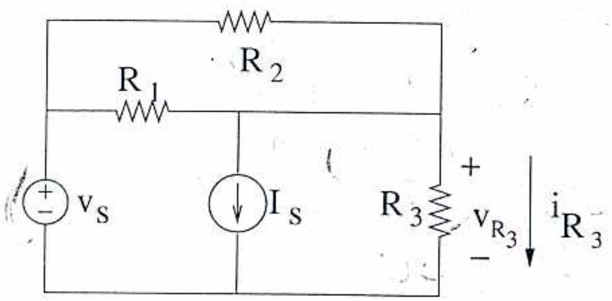
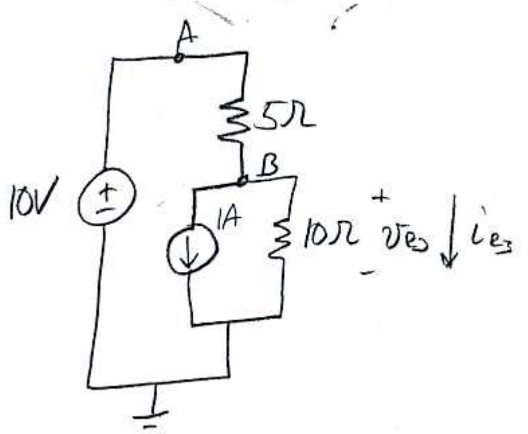
4 Suspiscious of superposition?

Given the following circuit:



Component	Value
V_s	10 V
I_s	1 A
R_1	10 Ω
R_2	10 Ω
R_3	10 Ω

(a) Clearly demonstrate the use of **node voltage analysis, without superposition,** to compute the voltage drop across R_3 (v_{R_3}), the current through R_3 (i_{R_3}), and the power dissipated by R_3 (P_{R_3}). Box your answers here, and fill your values into the table in part (e). [6 points]



$$v_A = 10V$$

$$KCL_B: \frac{v_A - v_B}{5} = \frac{v_B}{10} + 1$$

$$\frac{1}{5}v_A - \frac{3}{10}v_B = 1$$

$$-\frac{3}{10}v_B = -1$$

$$v_B = -\frac{10}{3}V = \boxed{v_{R_3} = 3.33V}$$

$$i_{R_3} = \frac{v_{R_3}}{R_3} = \frac{-3.33V}{10\,\Omega} = \boxed{0.33A}$$

$$P_{R_3} = v_{R_3}\, i_{R_3} = \boxed{1.10W}$$

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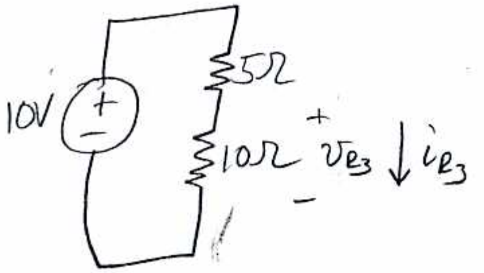
(b) Using an analysis method of your choice, compute the voltage drop across R_3 (v_{R_3}), the current through R_3 (i_{R_3}), and the power, dissipated by R_3 (P_{R_3}) if the current source is eliminated ($I_s = 0$). Box your answers here, and fill your values into the table in part (e). [6 points]

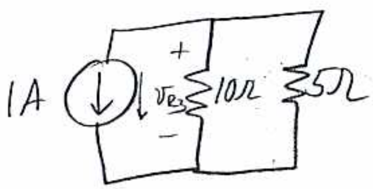


$$v_{R_3} = 10\,\frac{10}{15} = \boxed{6.67\text{V}}$$

$$i_{R_3} = \frac{6.67}{10} = \boxed{0.667\text{A}}$$

$$P_{R_3} = 6.67(0.667) = \boxed{4.45\text{W}}$$

(c) Using an analysis method of your choice, compute the voltage drop across R_3 (v_{R_3}), the current through R_3 (i_{R_3}), and the power dissipated by R_3 (P_{R_3}) if the voltage source is eliminated ($V_s = 0$) (and I_s is its original value of 1 A). Box your answers here, and fill your values into the table in part (e). [6 points]



$$i_{R_3} = -1\text{A}\,\frac{1/10}{1/10 + 1/5} = \boxed{-0.33\text{A}}$$

$$v_{R_3} = \boxed{-3.33\text{V}}$$

$$P_{R_3} = \boxed{1.10\text{W}}$$

(d) Using an analysis method of your choice, compute the voltage drop across R_3 (v_{R_3}), the current through R_3 (i_{R_3}), and the power dissipated by R_3 (P_{R_3}) if the current source is eliminated ($I_s = 0$) and $V_s = 20$ V (double the original voltage). Box your answers here, and fill your values into the table in part (e). [6 points]

$$\boxed{\begin{aligned} v_{R_3} &= 13.34\text{V} \\ i_{R_3} &= 1.33\text{A} \\ P_{R_3} &= 17.74\text{W} \end{aligned}}$$

(e) Notice what happened to v_{R_3}, i_{R_3} and P_{R_3} when V_s doubled with the current source eliminated (part (d) vs. part (b)). Given the linearity of the circuit and the principles of superposition, complete the following table for the voltage drop across R_3 (v_{R_3}), the current through R_3 (i_{R_3}), and the power dissipated by R_3 (P_{R_3}) for the two remaining combinations of voltage and current source values indicated in the table. You shouldn't have to perform any direct circuit analysis to complete this table, but please show your work. [6 points]

Part	V_s (V)	I_s (A)	v_{R_3} (V)	i_{R_3} (A)	P_{R_3} (W)
a	10	1	3.33	0.333	1.10
b	10	0	6.67	0.667	4.45
c	0	1	-3.33	-0.333	1.10
d	20	0	13.34	1.33	17.74
e	20	1	10.0	1.0	10.0
e	40	1	23.35	2.33	54.4

$$v_{R_3} = v_{R_3(d)} + v_{R_3(c)}$$
$$i_{R_3} = i_{R_3(d)} + i_{R_3(c)}$$
$$P_{R_3} = v_{R_3} i_{R_3}$$

$$v_{R_3} = 2v_{R_3(d)} + v_{R_3(c)}$$
$$i_{R_3} = 2i_{R_3(d)} + i_{R_3(c)}$$
$$P_{R_3} = v_{R_3} i_{R_3}$$